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S^1 -wrapped D3-branes on Conifolds

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ABSTRACT

We construct a D3-brane wrapped on S^1 , which is fibred over a resolved conifold. Whereas a fractional D3-brane on the resolved conifold is not supersymmetric and has a naked singularity, our solution is supersymmetric and regular everywhere. We also consider an S^1 -wrapped D3-brane on the resolved cone over $T^{1,1}/Z_2$, as well as on the deformed conifold. In the former case, we obtain a regular supergravity dual to a certain four-dimensional field theory whose Lorentz and conformal symmetries are broken in the IR region and restored in the UV limit.

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1 Introduction

D3-branes no doubt provide the most natural framework for the study of strongly-coupled four-dimensional Yang-Mills theory from the points of view of supergravity and string theory, *via* the AdS/CFT correspondence [1, 2, 3]. In order to reduce the supersymmetry to a minimum, one can replace the six-dimensional Euclidean transverse space with a Calabi-Yau manifold.¹ The simplest example of six-dimensional Calabi-Yau manifolds is the (non-compact and singular) conifold, defined as a cone over $T^{1,1} = (S^3 \times S^3)/S^1$. The near-horizon geometry of the D3-brane now becomes $\text{AdS}_5 \times T^{1,1}$, which provides a supergravity dual to the $\mathcal{N} = 1$, $D = 4$ superconformal Yang-Mills theory. There is a supersymmetric 2-cycle in the $T^{1,1}$ space, upon which one can wrap additional D5-branes or NS5-branes, giving rise to supersymmetric fractional D3-branes [5, 6, 7, 8]. The conformal symmetry is broken by the distance-dependent logarithmic contribution to the D3-brane charge. The solution, however, has a short-distance naked singularity and hence provides a structural behavior only at large distance, corresponding to the UV (ultra-violet) region of the dual field theory.

In the construction of the fractional D3-brane of type IIB theory, the complex 3-form $F_{(3)} = F_{(3)}^{\text{RR}} + i F_{(3)}^{\text{NS}}$ is set proportional to the complex self-dual 3-form $\omega_{(3)}$ of the conifold, such that it contributes non-trivially to the Bianchi identity of the 5-form: $dF_{(5)} = \frac{1}{2}i \bar{F}_{(3)} \wedge F_{(3)}$. To resolve the above naked singularity, ω_3 must be square integrable at short distance. This requires that the conifold must have non-collapsing 3-cycles [9]. There are two smoothed-out versions of the conifold, namely the deformed conifold and the resolved conifold [10]. In the former case, the singular apex is blown up to a smooth three-sphere, and hence it has a non-collapsing 3-cycle. The fractional D3-brane on the deformed conifold was constructed in [6]. This solution is supersymmetric and regular everywhere. On the other hand, in the case of the resolved conifold, the singular apex is blown up to a smooth two-sphere. Thus, it has a non-collapsing 2-cycle, but a collapsing 3-cycle. The fractional D3-brane over the resolved conifold was constructed in [11], and was shown to have a repulson-like naked singularity. Furthermore, it was shown to be non-supersymmetric [12, 13].

Since the resolved conifold has non-collapsing 2-cycles, the solution would be regular if it was the harmonic 2-form instead of the 3-form to provide the D3-brane source. Following the technique developed in [14], we consider the D3-brane wrapped on S^1 , which is fibred

¹An analogous construction was proposed earlier in [4] for the M2-brane.

over the resolved conifold. Besides the Kähler form, there are two additional harmonic 2-forms supported by the resolved conifold. One of them is square integrable at short distance and the resulting solution is regular everywhere. The harmonic 2-form carries non-trivial Taub-NUT type flux. Consequently, it is not normalizable at large distance, and contributes a distance-dependent D3-brane charge. The other 2-form falls off rapidly at infinity and does not carry any flux, which implies that the D3-brane charge is well-defined. However, it is not square integrable at short distance and hence the solution is singular in that region. We show that both solutions are supersymmetric, preserving the minimal amount of supersymmetry.

Recently, it was shown that the cone over $T^{1,1}/Z_2$ can also be resolved. The metric was obtained in [15, 16, 17]. It describes a complex-line bundle over $S^2 \times S^2$, with the topology $R^2 \times S^2 \times S^2$. In this case, the singular apex is blown up to a smooth $S^2 \times S^2$, and hence the manifold has a non-collapsing 4-cycle. On the other hand, there are no 4-cycles in $T^{1,1}/Z_2$. It follows that the 4-form does not carry non-trivial charge, and hence is fully normalizable. In six dimensions, a 4-form is Hodge dual to a 2-form. This enables us to construct a regular S^1 -wrapped D3-brane on the resolved cone over $T^{1,1}/Z_2$, which we show to be supersymmetric. In this configuration, the conformal and Lorentz symmetries of the original, unwrapped D3-brane are broken, but both are restored at large distance.

This paper is organized as follows. In section 2, we discuss the general construction of a D3-brane wrapped on S^1 , which is fibred over the six-dimensional transverse space. By T-dualizing and lifting the solution to eleven dimensions, we find the condition for which the supersymmetry is preserved. We show that the S^1 -wrapped D3-brane and fractional D3-brane have a common origin as the modified supermembrane in M-theory. In section 3, we consider the case in which the transverse space is a conifold. For this, a fractional D3-brane is singular, whereas an S^1 -wrapped D3-brane is regular everywhere. In sections 4 and 5, we find regular S^1 -wrapped D3-brane solutions on the resolved conifold over $T^{1,1}$ or $T^{1,1}/Z_2$, respectively. The latter solution is of particular interest since both conformal and Lorentz symmetries are broken in the IR region of the dual field theory, and restored in the UV limit. On the other hand, a D3-brane wrapped over S^1 on the deformed conifold has a naked singularity at short distance, as we see in section 6. We present conclusions in section 7.

2 S^1 wrapped D3-brane

The D3-brane of type IIB supergravity is supported by the self-dual 5-form field strength, with a six-dimensional Ricci-flat transverse space. Due to the Bianchi identity $dF_{(5)} = F_{(3)}^{\text{NS}} \wedge F_{(3)}^{\text{RR}}$, one can construct a fractional D3-brane if the transverse space has a self-dual 3-cycle. If instead the transverse space has a 2-cycle $L_{(2)}$, we can construct an S^1 -wrapped D3-brane with one of the world-volume coordinates fibred over the transverse space. Using the same technique developed in [14], we find that the solution is given by

$$\begin{aligned} ds_{10}^2 &= H^{-\frac{1}{2}} \left(-dt^2 + dx_1^2 + dx_2^2 + (dx_3 + \mathcal{A}_{(1)})^2 \right) + H^{\frac{1}{2}} ds_6^2, \\ F_{(5)} &= dt \wedge dx_1 \wedge dx_2 \wedge (dx_3 + \mathcal{A}_{(1)}) \wedge dH^{-1} - *_6 dH \\ &\quad + m *_6 L_{(2)} \wedge (dx_3 + \mathcal{A}_{(1)}) + H^{-1} dt \wedge dx_1 \wedge dx_2 \wedge L_{(2)}, \\ d\mathcal{A}_{(1)} &= m L_{(2)}, \end{aligned} \tag{1}$$

where $L_{(2)}$ is a harmonic 2-form in the transverse space of the metric ds_6^2 , and $*_6$ is the Hodge dual with respect to ds_6^2 . The equations of motion are satisfied, provided that

$$\square H = -\frac{1}{2} m^2 L_{(2)}^2, \tag{2}$$

where \square is the Laplacian in ds_6^2 .

A convenient method of determining the preserved supersymmetry of the solution is to T-dualize the fibre coordinate x_3 to obtain a modified D2-brane in type IIA theory and then dimensionally oxidize the solution to $D = 11$. The modified D2-brane is given by

$$\begin{aligned} ds_{10}^2 &= H^{-\frac{5}{8}} (-dt^2 + dx_1^2 + dx_2^2) + H^{\frac{3}{8}} (ds_6^2 + dz_1^2), \\ F_{(4)} &= dt \wedge dx_1 \wedge dx_2 \wedge dH^{-1} + *_6 L_{(2)}, \quad F_{(3)} = m L_{(2)} \wedge dz_1, \quad \phi = \frac{1}{4} \log(H). \end{aligned} \tag{3}$$

Note that, under T-duality, the fibre coordinate is untwisted and corresponds to the z_1 coordinate of the transverse space. Lifting to $D = 11$ yields the modified M2-brane, which is given by

$$\begin{aligned} ds_{11}^2 &= H^{-\frac{2}{3}} (-dt^2 + dx_1^2 + dx_2^2) + H^{\frac{1}{3}} (ds_6^2 + dz_1^2 + dz_2^2), \\ F_{(4)} &= dt \wedge dx_1 \wedge dx_2 \wedge dH^{-1} + m L_{(4)}, \end{aligned} \tag{4}$$

where

$$L_{(4)} = *_6 L_{(2)} + L_{(2)} \wedge dz_1 \wedge dz_2 \tag{5}$$

is a self-dual harmonic 4-form living in the 8-dimensional Ricci-flat transverse space with the metric $ds_6^2 + dz_1^2 + dz_2^2$. This type of modification to the M2-brane, which makes use

of the interaction in $d*F_{(4)} = \frac{1}{2}F_{(4)} \wedge F_{(4)}$, has been considered in [18, 19, 20, 12] (see also, *e.g.*, [21, 13, 22, 9, 23, 24, 25]). The introduction of $L_{(4)}$ to the M2-brane solution preserves all of the initial supersymmetries, provided that [20]

$$L_{abcd} \Gamma^{bcd} \epsilon = 0, \quad (6)$$

where ϵ is a Killing spinor in the transverse space.

Applying the same procedure to the fractional D3-brane yields a modified M2-brane with

$$L_{(4)} = G_{(3)} \wedge dz + \bar{L}_{(3)} \wedge d\bar{z}. \quad (7)$$

where $G_{(3)}$ is a complex self-dual 3-form in ds_6^2 and $z = z_1 + iz_2$. Thus, the wrapped D3-brane and fractional D3-brane can be united in a Spin(7) manifold with the metric $ds_6^2 + dz_1^2 + dz_2^2$ as its Gromov-Hausdorff limit.

3 On the conifold

The simplest Calabi-Yau manifold in $D = 6$ is the conifold $ds_6^2 = dr^2 + r^2 ds_{T^{1,1}}^2$, which is a Ricci-Flat cone over the Einstein space $T^{1,1}$

$$ds_{T^{1,1}}^2 = \lambda^2 (d\psi + \cos \theta d\phi - \cos \tilde{\theta} d\tilde{\phi})^2 + \frac{1}{6}(d\theta^2 + \sin^2 \theta d\phi^2) + \frac{1}{6}(d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\phi}^2). \quad (8)$$

The constant λ measures the squashing of the $U(1)$ fibre coordinate ψ . For the $T^{1,1}$ space to be Einstein, one must have $\lambda = 1/3$. The period of ψ is 4π . If instead the period is $4\pi/n$, the space is $T^{1,1}/Z_n$. There is a supersymmetric 2-cycle $\omega_{(2)}$ and its dual 3-cycle $\omega_{(3)}$, which are given by

$$\omega_{(2)} = \Omega_{(2)} + \tilde{\Omega}_{(2)}, \quad \omega_{(3)} = \frac{1}{3}(d\psi + \cos \theta d\phi - \cos \tilde{\theta} d\tilde{\phi}) \wedge (\Omega_{(2)} + \tilde{\Omega}_{(2)}), \quad (9)$$

where $\Omega_{(2)}$ and $\tilde{\Omega}_{(2)}$ are the volume-forms of the two S^2 . Thus, the conifold supports a harmonic 2-form and complex self-dual 3-form

$$L_{(2)} = \frac{1}{6}\omega_{(2)}, \quad G_{(3)} = \omega_{(3)} + i\omega_{(2)} \wedge \frac{dr}{r}. \quad (10)$$

There are two ways of modifying a D3-brane on a conifold. The first is to equate the complex 3-form field strength in type IIB supergravity to the above self-dual 3-form [5]. Since $\text{Re}(G_{(3)}) = \omega_{(3)}$ carries non-trivial flux, this describes an additional D5-brane wrapped on the supersymmetric 2-cycle, and hence it is called a fractional D3-brane. The modification to the harmonic function H is a logarithmic contribution from the 5-brane charge:

$$H = 1 + \frac{Q + 15m^2 \log r}{r^4}. \quad (11)$$

This case has been extensively studied. The solution has a naked singularity at small distance and resolutions have been proposed. As discussed in the introduction, there are two resolutions to the conifold itself, namely the deformed conifold and the resolved conifold [10]. In order to have regular small-distance behavior in H , it is clear that the 3-cycle should be non-collapsing. This is the case for the deformed conifold but not for the resolved conifold. Thus, there is no regular fractional D3-brane on the resolved conifold constructed so far [11]. In fact, the solution is non-supersymmetric [12, 13].

In this paper, we instead consider a wrapped D3-brane on S^1 which is fibred over the conifold, as described in (1). In this case, the function H is modified in power law of r :

$$H = 1 + \frac{m^2}{4r^2} + \frac{Q}{r^4}. \quad (12)$$

The solution is regular everywhere already. Even though the $1/r^4$ term dominates at small r , the conformal symmetry is broken everywhere due to the fibration which carries non-vanishing charge

$$\int_{r \rightarrow \infty} L_{(2)} = Q_2 \neq 0. \quad (13)$$

This charge is analogous to the Taub-NUT charge.

In addition to the $L_{(2)}$ given in (10), the conifold supports another harmonic 2-form, given by

$$\tilde{L}_{(2)} = \frac{2}{3r^5} dr \wedge (d\psi + \cos \theta d\phi - \cos \tilde{\theta} d\tilde{\phi}) + \frac{1}{6r^4} (\Omega_{(2)} - \tilde{\Omega}_{(2)}), \quad (14)$$

Now the function H is modified to

$$H = 1 + \frac{Q}{r^4} - \frac{\tilde{m}^2}{20r^{10}}. \quad (15)$$

This is very different from the above S^1 -wrapped D3-brane. The fibration does not carry any charge, namely

$$\int_{r \rightarrow \infty} \tilde{L}_{(2)} = 0. \quad (16)$$

In fact, the fibration rapidly vanishes at large r . As a consequence, the solution becomes $\text{AdS}_5 \times T^{1,1}$ at large r , and hence both the conformal and Lorentz symmetries are restored. It has a naked singularity at small but finite $r = r_0$. As we shall see in section 5, when the principal orbit is replaced with $T^{1,1}/Z_2$, this naked singularity can be resolved.

Finally there is the Kähler form. Its contribution to the function H is so badly behaved at large distance that there is an unresolvable naked singularity. Furthermore, as we shall see in section 4, it will break the supersymmetry. We shall not consider the Kähler form in this paper.

4 On the resolved conifold

The metric of the resolved conifold over $T^{1,1}$ is given by

$$ds_6^2 = d\rho^2 + a^2 (\Sigma_1^2 + \Sigma_2^2) + b^2 (\sigma_1^2 + \sigma_2^2) + c^2 (\Sigma_3 - \sigma_3)^2. \quad (17)$$

The functions a , b and c depend only on the radial variable ρ ; σ_i and Σ_i are left-invariant 1-forms of $SU(2) \times SU(2)$. They can be expressed in terms of Euler angles as

$$\begin{aligned} \sigma_1 + i\sigma_2 &= e^{-i\psi} (d\theta + i \sin \theta d\phi), & \sigma_3 &= d\psi + \cos \theta d\phi, \\ \Sigma_1 + i\Sigma_2 &= e^{-i\tilde{\psi}} (d\tilde{\theta} + i \sin \tilde{\theta} d\tilde{\phi}), & \Sigma_3 &= d\tilde{\psi} + \cos \tilde{\theta} d\tilde{\phi}, \end{aligned} \quad (18)$$

and they satisfy

$$d\sigma_i = -\frac{1}{2}\epsilon_{ijk} \sigma_j \wedge \sigma_k, \quad d\Sigma_i = -\frac{1}{2}\epsilon_{ijk} \Sigma_j \wedge \Sigma_k. \quad (19)$$

Note that, although there are ostensibly six coordinates here, when one substitutes them into (17), ψ and $\tilde{\psi}$ appear only through the combination $\psi - \tilde{\psi}$.

The existence of Killing spinors implies that functions a , b and c satisfy the following first-order equations:

$$2a \dot{a} = c = 2b \dot{b}, \quad \dot{c} = 1 - \frac{c^2}{2a^2} - \frac{c^2}{2b^2}. \quad (20)$$

Here, a dot denotes a derivative with respect to ρ . The solution for the resolved conifold is given by

$$\begin{aligned} a^2 &= \frac{1}{6}r^2, & b^2 &= \frac{1}{6}(r^2 + 6\ell^2), & c^2 &= \frac{r^2}{9h^2}, \\ h^2 &= \frac{r^2 + 6\ell^2}{r^2 + 9\ell^2}, & d\rho &= h dr, \end{aligned} \quad (21)$$

where we have introduced a more convenient radial variable r , which runs from 0 to ∞ . As r approaches 0, the metric becomes $R^4 \times S^2$, which is the topology of the manifold. Asymptotically at large distance, the metric becomes a cone over $T^{1,1}$.

We are interested in finding a harmonic 2-form supported by this metric. The most general ansatz for a 2-form with respect to the isometry of (17) is given by

$$L_{(2)} = u_1 e^0 \wedge e^3 + u_2 e^1 \wedge e^2 + u_3 e^4 \wedge e^5, \quad (22)$$

expressed in the vielbein basis $e^0 = h dr$, $e^1 = a \Sigma_1$, $e^2 = a \Sigma_2$, $e^3 = c (\Sigma_3 - \sigma_3)$, $e^4 = b \sigma_1$ and $e^5 = b \sigma_2$. The closure and co-closure of $L_{(2)}$ yield the following solution:

$$u_1 = -c_3 - \frac{6\ell^2 c_1}{(r^2 + 6\ell^2)^2} + \frac{2(r^2 + 3\ell^2) c_2}{r^4 (r^2 + 6\ell^2)},$$

$$\begin{aligned}
u_2 &= c_3 + \frac{c_1}{r^2 + 6\ell^2} + \frac{c_2}{r^4 (r^2 + 6\ell^2)} , \\
u_3 &= -c_3 + \frac{(r^2 + 12\ell^2) c_1}{(r^2 + 6\ell^2)^2} - \frac{c_2}{r^2 (r^2 + 6\ell^2)^2} .
\end{aligned} \tag{23}$$

Clearly, the harmonic 2-form associated with the c_3 terms is the Kähler form. The one associated with the c_1 terms, given by

$$L_{(2)} = -\frac{6\ell^2}{(r^2 + 6\ell^2)^2} e^0 \wedge e^3 + \frac{1}{r^2 + 6\ell^2} e^1 \wedge e^2 + \frac{r^2 + 12\ell^2}{(r^2 + 6\ell^2)^2} e^4 \wedge e^5, \tag{24}$$

gives rise to the $L_{(2)}$ in (10) at large r . Hence it has a non-trivial flux. The square of this form is

$$L_{(2)}^2 = \frac{4(r^4 + 18\ell^2 r^2 + 108\ell^4)}{(r^2 + 6\ell^2)^4}. \tag{25}$$

Thus, it is square integrable for $r \rightarrow 0$ but not normalizable at large distance. There exists a regular solution to H in (2) which, after choosing appropriate integration constants, is given by

$$H = 1 + \frac{m^2}{4(r^2 + 6\ell^2)}. \tag{26}$$

The solution is regular everywhere. At small distance $r \rightarrow 0$, the function H is a constant and the metric has the topology of $M_3 \times R^4 \times S^3 \times S^1$. At large distance, H behaves like (12).

The 2-form associated with the c_2 terms in (23) is given by

$$\tilde{L}_{(2)} = \frac{2(r^2 + 3\ell^2)}{r^4 (r^2 + 6\ell^2)^2} e^0 \wedge e^3 + \frac{1}{r^4 (r^2 + 6\ell^2)} e^1 \wedge e^2 - \frac{1}{r^2 (r^2 + 6\ell^2)^2} e^4 \wedge e^5. \tag{27}$$

This gives rise to (14) at large r . We find that

$$\tilde{L}_{(2)}^2 = \frac{12(r^4 + 6\ell^2 r^2 + 12\ell^4)}{r^8 (r^2 + 6\ell^2)^4}, \tag{28}$$

which falls off rapidly at large distance but is not square integrable at small distance. The resulting wrapped D3-brane solution has a naked singularity.

Both of the wrapped D3-branes are supersymmetric. To see this we first note that, after setting $c_3 = 0$, the three u_i 's satisfy the following linear relation:

$$u_1 - u_2 + u_3 = 0. \tag{29}$$

This linear dependence is crucial for the preservation of supersymmetry. To demonstrate this, we perform T-duality on the wrapped coordinate and lift the solution to $D = 11$, as discussed in section 2. It is straightforward to verify that the supersymmetric condition (6)

precisely implies (29). It is instructive to examine the case of the fractional D3-brane, for which the complex self-dual harmonic 3-form is given by $G_{(3)} = L_{(3)} + i*_6 L_{(3)}$, where

$$L_{(3)} = \frac{1}{c a^2} e^3 \wedge e^1 \wedge e^2 + \frac{1}{c b^2} e^3 \wedge e^4 \wedge e^5. \quad (30)$$

In this case, the vielbein components of $G_{(3)}$ are not linearly dependent, except at $r = \infty$. Thus, as shown in [12, 13], this self-dual 3-form cannot satisfy the supersymmetric condition

$$L_{abc} \Gamma^{abc} = 0, \quad (*_6 L)_{abc} \Gamma^{abc} = 0, \quad (31)$$

obtained in [8].

It should be emphasized that a linear dependency of the vielbein components of a harmonic form is only a necessary condition, but not sufficient. Clearly, the vielbein components of the Kähler form are linearly dependent since they are constants; however, the resulting solution is not supersymmetric since the specific relationship (6) is not satisfied.

In this section we have found that, due to the existence of the non-collapsing 2-cycle in the resolved conifold, there exists a square-integrable harmonic 2-form at short distance. This yields a regular and supersymmetric S^1 -wrapped D3-brane.

5 On the resolved cone over $T^{1,1}/Z_2$

The first-order equations (20) admit a more general regular solution:

$$\begin{aligned} a^2 &= \frac{1}{12} (r^2 + \ell_1^2), & b^2 &= \frac{1}{12} (r^2 + \ell_2^2), & c^2 &= \frac{r^2}{36h^2}, \\ h^2 &= \frac{(r^2 + \ell_1^2)(r^2 + \ell_2^2)}{2r^4 + 3(\ell_1^2 + \ell_2^2)r^2 + 6\ell_1^2 \ell_2^2}, & d\rho &= h dr. \end{aligned} \quad (32)$$

This solution, a more general version of the metric with $\ell_1 = \ell_2$ obtained in [15, 16], was constructed in [17] in a different coordinate system. The radial coordinate runs from 0 to ∞ , with the geometry of $R^2 \times S^2 \times S^2$ at small distance and the cone over $T^{1,1}/Z_2$ asymptotically. The metric describes a complex-line bundle over $S^2 \times S^2$. Since the ψ in (8) becomes the circular coordinate of R^2 as $r \rightarrow 0$, it has a period of 2π . Thus, the principal orbit is a $T^{1,1}/Z_2$ instead of the $T^{1,1}$ space of the resolved conifold. Although it may appear that the metric reduces to a resolved conifold if one of the ℓ_i vanishes, this is not the case since they have rather different principal orbits.

In this case there are, once again, three harmonic 2-forms. As in the previous case, we shall not consider the Kähler form. The one that carries non-trivial flux is given by (22)

with

$$u_1 = \frac{(\ell_1^2 - \ell_2^2)(r^4 - \ell_1^2 \ell_2^2)}{(r^2 + \ell_1^2)^2(r^2 + \ell_2^2)^2}, \quad u_2 = \frac{r^4 + 2\ell_1^2 r^2 + \ell_1^2 \ell_2^2}{(r^2 + \ell_1^2)^2(r + \ell_2^2)}, \quad u_3 = \frac{r^4 + 2\ell_2^2 r^2 + \ell_1^2 \ell_2^2}{(r^2 + \ell_1^2)(r + \ell_2^2)^2}. \quad (33)$$

It is straightforward to verify that the 2-form is square integrable at short distance but non-normalizable at large distance. The function H is now given by

$$H = 1 + \frac{m^2 (\ell_1^2 - \ell_2^2)^2 (4r^2 + \ell_1^2 + \ell_2^2)}{4(\ell_1^2 - 3\ell_2^2)(3\ell_1^2 - \ell_2^2)(r^2 + \ell_1^2)(r^2 + \ell_2^2)} \quad (34)$$

$$+ \frac{m^2 (\ell_1^2 + \ell_2^2)}{2\sqrt{3}((\ell_1^2 - 3\ell_2^2)(3\ell_1^2 - \ell_2^2))^{3/2}} \log \left(\frac{4r^2 + 3\ell_1^2 + 3\ell_2^2 - \sqrt{3(\ell_1^2 - 3\ell_2^2)(3\ell_1^2 - \ell_2^2)}}{4r^2 + 3\ell_1^2 + 3\ell_2^2 + \sqrt{3(\ell_1^2 - 3\ell_2^2)(3\ell_1^2 - \ell_2^2)}} \right).$$

The solution is regular everywhere, with H as a positive constant at $r = 0$ and behaving like (12) at large r . It is worth mentioning that, although the u_i in (33) reduce to the previous case when one of the ℓ_i vanishes, the same does not hold for the function H . This is because, in each case, the H presented is not the most general solution but rather has one of the integration constants chosen such that the solution is regular. This does not commute with setting ℓ_i equal to zero. As emphasized earlier, it does not come as a surprise that the resolved conifold (over $T^{1,1}$) cannot be obtained from the resolved cone on $T^{1,1}/Z_2$ in the limit of vanishing ℓ_i .

The other harmonic 2-form has vanishing flux, given by

$$u_1 = \frac{(2r^2 + \ell_1^2 + \ell_2^2)}{(r^2 + \ell_1^2)^2(r^2 + \ell_2^2)^2}, \quad u_2 = \frac{1}{(r^2 + \ell_1^2)^2(r^2 + \ell_2^2)}, \quad u_3 = -\frac{1}{(r^2 + \ell_1^2)(r^2 + \ell_2^2)^2}. \quad (35)$$

In this case, we find that the 2-form is normalizable:

$$\int_0^\infty \sqrt{g} L_{(2)}^2 = \frac{\ell_1^2 + \ell_2^2}{864 \ell_1^4 \ell_2^4}, \quad (36)$$

ensuring that the function H is well-behaved at both large and small r . This function is given by

$$H = 1 + \frac{\tilde{m}^2 ((\ell_1^2 - \ell_2^2)^2 r^2 + (\ell_1^2 + \ell_2^2)((\ell_1^2 - \ell_2^2)^2 - \ell_1^2 \ell_2^2))}{4\ell_1^4 \ell_2^4 (\ell_1^2 - 3\ell_2^2)(3\ell_1^2 - \ell_2^2)(r^2 + \ell_1^2)(r^2 + \ell_2^2)} \quad (37)$$

$$+ \frac{\tilde{m}^2 (\ell_1^2 - \ell_2^2)}{2\sqrt{3} \ell_1^4 \ell_2^4 ((\ell_1^2 - 3\ell_2^2)(3\ell_1^2 - \ell_2^2))^{3/2}} \log \left(\frac{4r^2 + 3\ell_1^2 + 3\ell_2^2 - \sqrt{3(\ell_1^2 - 3\ell_2^2)(3\ell_1^2 - \ell_2^2)}}{4r^2 + 3\ell_1^2 + 3\ell_2^2 + \sqrt{3(\ell_1^2 - 3\ell_2^2)(3\ell_1^2 - \ell_2^2)}} \right).$$

Thus, we see that H is regular everywhere; it is constant at small distance and behaves like (15) at large distance. This solution is of particular interest. The metric interpolates a product space of M_3 and $U(1)$ bundle over $R^2 \times S^2 \times S^2$ at short distance to $\text{AdS}_5 \times T^{1,1}/Z_2$ at large distance. This implies that, for the dual field theory, both conformal and Lorentz symmetries are broken in general but are restored in the UV limit.

The existence of a normalizable harmonic 2-form in this manifold can be understood by the following. In the resolved cone over $T^{1,1}/Z_2$, the original singular apex is blown up to a smooth $S^2 \times S^2$, implying a non-collapsing 4-cycle. Since there is no supersymmetric 4-cycle in $T^{1,1}/Z_2$, it follows that there can exist a normalizable harmonic 4-form. In six dimensions, a 4-form is Hodge dual to a 2-form, which we used to construct the wrapped D3-brane.

Both solutions are supersymmetric, since the functions u_i 's in both cases satisfy the supersymmetric condition (6).

As we have shown that in the resolved conifold (and also in the resolved cone over $T^{1,1}/Z_2$), there are two harmonic 2-forms. Since the two harmonic 2-forms have the same structure (22) and both satisfy the supersymmetric condition (17), they can be linearly superposed to give rise to a more general S^1 -wrapped D3-brane. It is also possible that the D3-brane wraps on a two-torus with the coordinates x_2 and x_3 fibred over the two 2-forms respectively.

6 On the deformed conifold

There is an alternative resolution to the singular conifold, called the deformed conifold [10]. The corresponding metric is given by

$$ds_6^2 = d\rho^2 + a^2 \left((\Sigma_1 + \sigma_1)^2 + (\Sigma_2 + \sigma_2)^2 \right) + b^2 \left((\Sigma_1 - \sigma_1)^2 + (\Sigma_2 - \sigma_2)^2 \right) + c^2 (\Sigma_3 - \sigma_3)^2, \quad (38)$$

where a , b and c are functions only of the radial variable ρ . They satisfy the first-order equations

$$\dot{a} = \frac{b^2 + c^2 - a^2}{4bc}, \quad \dot{b} = \frac{a^2 + c^2 - b^2}{4ac}, \quad \dot{c} = \frac{a^2 + b^2 - c^2}{2ab}. \quad (39)$$

There is only one regular solution, given by

$$\begin{aligned} a^2 &= \frac{1}{2}K \cosh^2(\frac{1}{2}r), & b^2 &= \frac{1}{2}K \sinh^2(\frac{1}{2}r), & c^2 &= \frac{1}{3K^2}, \\ K &= \frac{(\sinh(2r) - 2r)^{1/3}}{2^{1/3} \sinh r}, & h^2 &= \frac{1}{3K^2}, & d\rho &= h dr. \end{aligned} \quad (40)$$

Now there exist only two harmonic 2-forms. One is the Kähler form

$$J = e^0 \wedge e^3 - e^1 \wedge e^5 + e^2 \wedge e^4. \quad (41)$$

The other harmonic 2-form is given by

$$L_{(2)} = \frac{1}{\sinh(2r) - 2r} (e^0 \wedge e^3 + \frac{1}{2}e^1 \wedge e^5 - \frac{1}{2}e^2 \wedge e^4), \quad (42)$$

which behaves like (14) at large distance but is not square integrable at small distance. This is to be expected, since the deformed conifold does not have any non-collapsing 2-cycles or 4-cycles. The function H behaves like (15) at large distance but the metric has a naked singularity at short distance.

Note that, for (42), the supersymmetric condition (6) is still satisfied and hence our solution is supersymmetric, albeit the naked singularity.

Since the deformed conifold has a non-collapsing 3-cycle, the natural resolution is that of a fractional D3-brane, supported by a self-dual harmonic 3-form. Indeed, the fractional D3-brane on the deformed conifold is regular and supersymmetric [6].

7 Conclusions

Since the conifold supports both harmonic complex self-dual 3-forms and 2-forms, there are two ways to add an additional flux contribution to the D3-brane. The first construction is to utilize the 3-form, which gives rise to fractional D3-branes. In this paper, we consider the second possibility, which utilizes the 2-form by wrapping the D3-brane on S^1 , which is fibred over the conifold. The deformed conifold has a non-collapsing supersymmetric 3-cycle and, consequently, the fractional D3-brane is regular, whereas the wrapped D3-brane has a singularity at small distance. On the other hand, the resolved conifold has a non-collapsing, supersymmetric 2-cycle. Thus, the fractional D3-brane is singular, whilst the wrapped D3-brane is regular. In both deformed and resolved conifolds, the 2-forms and 3-forms are not normalizable, due to the integrability at either large or small distance.

We also consider the resolved cone over $T^{1,1}/Z_2$. In this case, the apex singularity is blown up to a smooth $S^2 \times S^2$. Consequently, the manifold has a normalizable harmonic 4-form. In the six-dimensional transverse space, the 4-form is Hodge dual to a 2-form, which we use to construct a wrapped D3-brane. The resulting solution is regular everywhere, interpolating a product space of M_3 and $U(1)$ bundle over $R^2 \times S^2 \times S^2$ at short distance to $\text{AdS}_5 \times S^5$ at large distance. From the viewpoint of the dual field theory, this implies that the broken conformal and Lorentz symmetries are both restored in the UV limit.

We argue that the seemingly different fractional D3-brane and S^1 -wrapped D3-brane can be united by T-duality as the same regular, modified M2-brane on a $\text{Spin}(7)$ manifold, which gives rise to the resolved and deformed conifolds in different Gromov-Hausdorff limits. Recent results for the G_2 unification of resolved and deformed conifolds [26, 27, 28] strengthens the argument. Thus, from the M-theory viewpoint, the fractional D3-branes

are natural for the deformed conifold, whilst the wrapped D3-branes constructed in this paper are natural for the resolved conifold.

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